Using Recursion in Models and Decision Making: Recursion Using Rate of Change

IV.C Student Activity Sheet 5: Newton's Law of Cooling

Have you ever noticed that a container of cold liquid, such as a glass of iced tea, creates condensation on the outside of the container? Or that a cup of hot coffee does not always stay hot?

What happened to the temperatures of those cups of liquid? In this activity, you will investigate changes in the temperature of a liquid over time.

1. Suppose your teacher poured a cup of hot coffee at the beginning of class, set it on her desk, and then forgot about it. What would happen to the temperature of the coffee over time? Why do you think this is so?

2. Sketch a graph of the coffee's temperature over time.

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3. Collect a set of data showing the temperature (degrees Fahrenheit) of a cup of hot liquid, such as coffee, changing over time (minutes). Then complete the column that asks for the difference between the temperature of the liquid and the room temperature. The remaining columns will be used in the next question.

Time (t)	Temperature of Liquid (L)	Temperature of Liquid (L) – Room Temperature (T)	First Differences (ΔT)	Successive Ratios

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4. Complete the table by computing the first differences and successive ratios. Use your calculator or spreadsheet to help with the computation.

5. Is the relationship between time and temperature linear or exponential? How do you know?

6. Use the information in the table to build a recursive rule for the difference in temperature from room temperature for each successive temperature reading.

7. What does the constant in the recursive rule represent?

8. Use your graphing calculator to make a scatterplot of temperature versus time. Sketch your results.

9. How does your scatterplot compare to the graph you sketched at the beginning? Explain any differences.

10. How does your scatterplot support the type of relationship you chose in Question 5?

11. The general form for an exponential function is $y = a(b)^x$, where a represents the initial condition and b represents the successive ratio, or base of the exponential function. Using data from your table, write a function rule to describe the temperature of the coffee (y) as a function of time (x).

12. What do the constants in the function rule represent?

13. Graph the function rule over the data in your scatterplot. Sketch your results.

14. **REFLECTION:** Compare the recursive rule and explicit function rule that you wrote in the previous questions. What do you notice?

    If you repeated this experiment in a room that was much cooler, what changes in your data would you expect? Why do you think so?

15. What would a scatterplot of the change in temperature (ΔT) versus the difference between the liquid's temperature and the room temperature (T) look like? Sketch your prediction, if needed.

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16. Use your graphing calculator to make a scatterplot of the change in temperature (ΔT) versus the difference between the liquid's temperature and the room temperature (T). Sketch your graph.

How does your graph compare to your prediction?

What kind of function appears to model your graph?

17. EXTENSION: Use an appropriate regression routine to find a function rule to model your scatterplot of ΔT versus T. Record your function rule, rounding to an appropriate number of places.

Graph the function rule over your scatterplot. How well does it fit the data?

Recall that a proportional relationship between the independent and dependent variables satisfies three criteria:

- The graph is linear and passes through the origin.
- The function rule is of the form $y = kx$.
- The ratio $\frac{y}{x}$ is constant for all corresponding values of x and y.

18. EXTENSION: Is your function rule from Question 17 a proportional relationship? Defend your answer using all three criteria.

Graph:

Function Rule:

Table:

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19. **REFLECTION:** Suppose the relationship between the change in temperature and the difference between the liquid's temperature and the ambient temperature is proportional. Write a proportionality statement to show the relationship between the two variables.

<table>
<tr><td>

</td><td>is proportional to</td><td>

</td></tr>
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20. **EXTENSION:** Choose a simple exponential function to validate the proportionality according to all criteria.

21. **EXTENSION:** Use the Internet to research other situations that could be modeled using a function from the same family as the one for Newton's Law of Cooling. Obtain a data set and generate a function model. Cite your sources. Support your choice of function model with mathematical reasoning. Make a prediction from your data set using your model. How reasonable is your prediction?